# Optimal Imprecision and Ignorance*

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**Abstract**

The information available to the capital market about a firm's investment is usually imprecise due to the nature of financial accounting measurements. In addition, management usually has an information advantage over the capital market regarding some aspects of investment, particularly the investment's inherent profitability. We refer to this information asymmetry as the market's ignorance. We study how such ignorance and measurement imprecision interact to simultaneously determine equilibrium investment and capital market prices. We find that if either imperfection is present, the other is desirable; otherwise, the resulting equilibrium is very inefficient. An appropriate balance of ignorance and imprecision induces investment decisions and prices that are close to first best. The greater the capital market's ignorance of profitability, the more imprecise should be the measurement of the firm's investment.

# 1 Introduction

Most accounting measurements are imprecise, providing at best a noisy representation of a firm's operations and their underlying economic events. Is such imprecision necessarily harmful, or can it be value enhancing? Is there an 'optimal' degree of imprecision for accounting measurements? If so, what are its key determinants? The answers to these questions may seem obvious, and indeed there seems to be a presumption among accounting policy makers that imprecision, *per se*, is undesirable. In this paper, we put this presumption to a rigorous test and obtain some surprising answers. We focus the analysis on imprecision in accounting measurements of a firm's investment. In practice, such measurements are notoriously imprecise because they are often based on subjective judgments and estimates and on simplistic conventions.

In the accounting, economics, and finance literature, there are currently two streams of thought that argue in favor of imprecision in public accounting disclosures. The first shows that public disclosure of information destroys risk-sharing opportunities while non-disclosure invites wasteful information gathering on private account, and the tradeoff between these two forces yields an optimal level of imprecision [Hirshleifer (1971), Diamond (1985), Verrecchia (1982)]. The second shows that disclosure imposes proprietary costs on a firm by informing competitors' actions, but disclosure mitigates skeptical beliefs in the capital market, making coarse disclosure optimal [Dye (1986), Gigler (1994), Verrecchia (1983)].

We show that there is an additional compelling demand for measurement imprecision that does not arise from risk-sharing or competitive considerations. In our paper, a demand for imprecision arises solely from a firm's concern for how its decisions are priced in the capital market. These prices depend upon the information directly available in accounting reports as well as upon inferences, about the hidden characteristics of the firm, drawn from these reports. These inferences have the potential of significantly changing the firm's decisions. Thus, imprecision in accounting reports simultaneously affects the firm's decisions and the pricing of those decisions in the capital market. In our model, this interaction of the firm's decisions with capital markets does not arise from any need to raise new capital, but rather with how the firm's ongoing operations are interpreted and priced.

A key assumption underlying our analysis is that at the time they choose the firm's investment, managers have superior information relative to the capital market. Given that in practice, managers expend enormous amounts of time and resources to collect and analyze information about alternative investment projects, it seems reasonable to assume that such 'ignorance' exists in the capital market. The ideal situation, leading to first-best investment and market prices, is one where neither ignorance nor measurement imprecision exists. In this sense, both ignorance and imprecision are undesirable; therefore, one may be tempted to conclude that the presence of either condition makes the elimination or minimization of the other more desirable. However, we show exactly the opposite is true. The presence of either one (ignorance or imprecision) without the other has disastrous consequences, while together they work reasonably well. Thus, if managers possess superior information about a project's profitability, then some degree of imprecision in the measurement of the firm's investment is highly desirable. Conversely, if measurement is imprecise, then some degree of ignorance in the capital market is highly desirable. Thus, there is an 'optimal' balance of measurement imprecision and ignorance for the capital market.

We study three informational regimes. In the first regime, measurement of investment is imprecise but the profitability of investment is publicly known (i.e., there is imprecision, but no ignorance). In this situation, we show that *any* imprecision in measurement, no matter how small, is equivalent to no measurement at all. The accounting measurement is completely ignored by the capital market. Firms rationally respond to this situation by investing myopically and are priced consistent with this myopic investment. The problem here is that the market knows too much! The market has sufficient information to solve the firm's investment problem, so it believes it can exactly anticipate the investment even though it cannot observe it. Thus, when the measurement does not coincide with the market's prior anticipations, the difference is attributed solely to the imprecision and is ignored. The measurement of the firm's investment needs to be infinitely precise to break this very bad equilibrium.

Empirically, imprecision in accounting measurement of investment is pervasive due to the judgmental nature of distinguishing between operating expenditures and assets. It is difficult to believe that these pervasive measurements are without information content. Therefore, the above result suggests that a setting in which capital market participants can perfectly anticipate the firm's investment – by solving its problem – is unrealistic. This motivates the study of the second informational regime where the firm knows more than the market about the profitability of its investment. We show that, given such ignorance, *perfect* measurement of investment also has disastrous consequences. Perfect measurement leads to perfect inferences of project profitability by the capital market resulting in a classical Spence (1974) type signaling equilibrium, leading to overinvestment. Parametric analysis reveals that the overinvestment could be so substantial that the firm's expected payoff is as little as 10% of the first-best.

In the third regime, we show that imprecision in accounting measurement, together with information asymmetry between the manager and the market, provides improvement over the preceding two bad equilibria. Information asymmetry (ignorance) is desirable because it allows imprecise measurements to have information content, thus eliminating the myopic investment problem. Imprecision in measurement is desirable because it alleviates the overinvestment problem associated with fully revealing signaling equilibria. Given both ignorance and imprecision, we show that the equilibrium involves 'noisy' signaling. We find conditions under which an appropriate level of imprecision actually restores the first-best investment schedule and achieves the first-best expected payoff to the firm. In this case, we are able to precisely characterize the optimal degree of imprecision and relate it to the critical exogenous variables. We obtain the surprising result that the greater the information asymmetry between the manager and the market, the *less* precise accounting measurement should be. Conversely, given some exogenous level of imprecision in the accounting measurement, there is an optimal degree of ignorance for the capital market; the greater the imprecision in accounting measurement, the greater should be the information asymmetry between the market and the firm's manager.

The above results may seem counter-intuitive. In trading economies, where the firm's investment decisions are suppressed and its cash flows are described by an *exogenous* stochastic process, imprecise information is better than no information, and barring risk sharing considerations, more precision is always better than less precision. This conventional wisdom fails when the interaction between firms' decision and the capital market's pricing of those decisions is made explicit. Our results are best understood in terms of an externality between two noisy signals where one signal reveals direct information about the firm's 'type' and the other (the accounting) signal reveals direct information about the firm's decision. As the first signal becomes more informative, the value of the second signal declines because the market relies more on its prior beliefs regarding the firm's decisions. At the limit, when the firm's type is perfectly known the accounting signal becomes worthless. Conversely, when the market obtains very precise information about the firm's decision, the market tends to ignore the signal on type, relying more on indirect inferences made from observation of the firm's decision. Unless both signals can be made infinitely precise, there is an optimal balance in the precision of the two signals.

Our result that noisy signals of endogenous actions are uninformative is closely related to the results in Bebchuk and Stole (1993), Bagwell (1995) and Kanodia and Mukherji (1996).[1] Bebchuk and Stole show that the capital market does not extract information about a firm's endogenous allocation of funds between short-term and long-term projects from the observable return to the short-term project. Bagwell established that in a leader-follower oligopoly, the leader's first mover advantage is completely destroyed if observation of the leader's output is noisy. Kanodia and Mukherji established that noisy separation of a firm's investment from its operating expenditures could only provide information on the firm's operating profits but could not provide any information on the firm's investment. Maggi (1999) extended Bagwell's analysis by showing that if the leader's output is based on private information, then noisy signals on that output are indeed informative. Maggi further established that in some cases there is a critical degree of noise that would fully restore the first-mover advantage of the leader. This result is similar to our result on the optimality of noise in the measurement of a firm's investment. However, Maggi explicitly avoids signaling considerations by having the leader privately observe a parameter that is not directly relevant to the follower. Noisy signaling lies at the heart of our analysis, because the private information on which the firm's investment is based is essential to the pricing of the firm in the capital market.

The literature on noisy signaling is sparse. Methodologically, the paper that is closest to our work is the Matthews and Mirman (1983) study of entry deterrence with limit pricing. In Matthews and Mirman, an incumbent producer, with private knowledge of an industry demand parameter, chooses an output level that stochastically affects the equilibrium price

[1] See also Fudenberg and Tirole (1986), Narayanan (1985), and Stein (1989).

in the commodity market. A potential entrant extracts information from the observed price and decides whether or not to enter. Unlike our work, their analysis is considerably simplified by the binary nature of the entrant's decision. Additionally, Matthews and Mirman do not provide any insights into an 'optimal' degree of noise.

The paper is organized as follows. In Section 2, we describe a benchmark model of the firm's investment decision under full information. This characterization illustrates the two-way interaction between investment and capital market pricing that is exploited in the rest of the paper. In Section 3, we study this interaction when accounting measurement of investment is imprecise and there is no other information asymmetry between the market and the firm's manager. In Section 4, we introduce asymmetric information (ignorance) about the profitability of investment and examine the consequences of perfect measurement of investment. Section 5 characterizes noisy signaling equilibria when both ignorance and imprecision are present. In Section 6, we specialize the model and derive our results on the optimality of imprecision and ignorance. Section 7 concludes.

## 2 Investment with Complete Information: A Benchmark Model

Consider the investment problem of a firm that is traded in a capital market. The firm's investment yields short-term and long-term returns. Short-term returns are consumed directly by the firm's current shareholders, but long-term returns are consumed through the pricing of the firm in the capital market. Investment of $k$ units yields a short term return of $\theta k - \frac{ck^2}{2}$, where the parameter $\theta$ is a summary statistic representing the profitability of the project in which the firm invests, and $\frac{ck^2}{2}$ is the cost of investment which is increasing and strictly convex. The profitability parameter is drawn from a distribution with density function $h(\theta)$. In all the settings we consider, the firm's manager is assumed to observe the parameter $\theta$ before choosing the firm's investment, and all agents in the economy are assumed risk neutral.

Initially, suppose that the capital market has full information, i.e. $\theta$ is common knowledge and the firm's investment is perfectly and directly observed. The firm's currently chosen investment $k$ and its current profitability $\theta$ affect (perhaps stochastically) the long-term returns generated by the firm. Hence its price in the capital market is described by some function $v(k, \theta)$. This pricing rule for the complete information setting is exogenous. We assume that $v(k, \theta)$ satisfies $v_k > 0$, $v_{kk} \leq 0$, $v_\theta > 0$, and $v_{k\theta} \geq 0$. Assuming that the firm invests to maximize the expected payoff to its current shareholders, the firm's problem is described by:

$$\max_k \quad \theta k - \frac{ck^2}{2} + v(k, \theta)$$

The firm's optimal investment schedule is described by the first order condition:

$$k_{FB}(\theta) = \frac{\theta + v_k(k, \theta)}{c} \tag{1}$$

where the subscript '$FB$ ' denotes first best.

The investment model described above captures, in a simple way, the two-way interaction between a firm's investment and its capital market price. Not only does the firm's investment affect its capital market value, as described by $v(k, \theta)$, but also the market's price response affects the firm's choice of investment, as indicated in (1). The firm's investment policy characterized in (1) is consistent with the 'net present value rule' which formally requires the firm to discount its expected future cash flows at an appropriate cost of capital. The market value $v(k, \theta)$ is, *in fact*, the present value obtained from the distribution of future cash flows and appropriately reflects both the market's assessment of this distribution and the time preferences of investors in the capital market. We do not explicitly use discount factors to derive the firm's value, since it is unlikely that equilibrium discount factors (cost of capital) would be independent of the market's beliefs and the level of the firm's investment. The firm's investment policy described in (1) indicates that the firm invests till the point where the marginal cost of investment equals the sum of the marginal short term return to investment and the marginal effect of investment on the value assigned by the capital market to the distribution of long term returns. Modeling the firm's investment problem in this fashion, allows us to study how accounting measurements and disclosures would affect the firm's investment choices through their interaction with capital markets.

The valuation rule, $v(k, \theta)$, exogenous to our analysis, really derives from a complex intertemporal equilibrium (see Kanodia (1980)). In such an intertemporal model the profitability parameter $\theta$ could evolve stochastically over time and the firm could have opportunities for new investment at every point in time. If the firm's current profitability $\theta$ affects the distribution of future profitability and, if either the firm's current investment directly affects the distribution of future cash flows or indirectly affects that distribution by constraining future investment opportunities, then the current value of the firm would indeed be a function of current investment $k$ and current profitability $\theta$. The assumptions we have specified for this exogenous value $v(k, \theta)$ would very likely be satisfied in such an intertemporal model where $v(k, \theta)$ is derived endogenously.

## 3 Imprecision Without Ignorance: A Myopia Result

The first-best setting assumes that the firm's investment is directly and perfectly observed by the capital market. In practice, however, investment is measured and reported by the financial accounting process. Accounting measurements of investment rely on numerous subjective judgments and estimates made by accountants and auditors. These are often determined by the application of simplistic conventions that induce classification errors (that are not uniform across firms and industries). Therefore, accounting reports (on a firm's investment) are necessarily imprecise. Below, we examine the effect of such imprecision on capital market prices and the firm's investment policy in the absence of ignorance about the firm's profitability parameter $\theta$. We show that imprecision without ignorance results in a very bad equilibrium.

As in the first-best economy, assume that the profitability parameter $\theta$ is common knowledge, but, contrary to first-best, assume that the firm's investment is not directly observed by the capital market. Instead, the firm's investment is imprecisely measured by an accounting system. Let $\tilde{s}$ denote the accounting report. Since the accounting measurement is stochastically related to the firm's actual investment, we model $\tilde{s}$ as a drawing from a distribution $F(s|k)$ parameterized by the *true* level of investment. At this point in the analysis, we assume only that $F$ has density $f(s|k)$ and fixed support $[\underline{s}, \ \overline{s}\ ]$.

The pricing rule in the capital market can depend only upon observable variables: the known parameter, $\theta$, and the imprecise accounting measurement, $s$. Therefore the equilibrium price in the capital market is some function $\varphi(s, \theta)$.

**Definition 1** *A pure strategy equilibrium consists of two schedules, an investment schedule,* $k(\theta)$, *and a pricing schedule,* $\varphi(s, \theta)$, *satisfying:*

*(i) Given $\varphi(s,\theta)$, $k(\theta)$ is optimal for the firm, i.e., for each $\theta$, $k(\theta)$ solves*

$$\max_{k} \quad \theta k - \frac{ck^2}{2} + \int_{\underline{s}}^{\overline{s}} \varphi(s,\theta)\, f(s|k)\, ds. \tag{2}$$

*(ii) $\varphi(s,\theta)$ incorporates beliefs that are consistent with the equilibrium investment schedule $k(\theta)$ and the firm's 'intrinsic value' $v(k,\theta)$.*

Condition (ii) of the equilibrium is a rational expectations (market efficiency) requirement, which is elaborated below.

We take a constructive approach to characterizing the equilibrium. First, we show that the equilibrium market price cannot depend on the accounting measure, $\widetilde{s}$. We then show that the firm's equilibrium investment must be myopic.

To see that the accounting signal does not affect the equilibrium pricing rule, suppose, to the contrary, that it does affect it. Suppose the pricing rule, $\varphi(s,\theta)$, is differentiable in $s$. The firm's optimization problem, as described in (2), can be re-written as:

$$\max_{k} \quad \theta k - \frac{ck^2}{2} + \varphi(\overline{s},\theta) - \int_{\underline{s}}^{\overline{s}} \varphi_s(s,\theta)\, F(s|k)\, ds.$$

The firm's optimal investment is characterized by the first-order condition:

$$\theta - ck - \int_{\underline{s}}^{\overline{s}} \varphi_s(s,\theta)\, F_k(s|k)\, ds = 0. \tag{3}$$

If $\varphi_s(s,\theta) > 0$, it may appear from (2) and (3) that accounting imprecision would have only a minor effect on the firm's investment, since the marginal effect of investment on the firm's value, as described in (1), has been replaced by its expectation. However, as yet we have said nothing about how $\varphi(s,\theta)$ is determined in equilibrium.

Let $k_M(\theta)$ be the solution to (3). Rationality of beliefs would imply:

$$\varphi(s,\theta) = E[v(k_M(\theta),\theta)|s] \tag{4}$$

Now, since the capital market understands the structure of the firm's problem, i.e. the market knows that the firm's investment is a solution to (3), the market must know the firm's investment policy. Given that the market additionally knows the parameter $\theta$, the market believes it knows the firm's investment, even though the actual investment chosen by the firm is not observed by the market. Since the market knows $\theta$ and believes it knows the firm's investment, the conditional expectation in (4) is vacuous. Given its beliefs the market must value the firm at

$$\varphi(s,\theta) = v(k_M(\theta),\theta),\ \forall s. \tag{5}$$

This implies that the equilibrium price in the market does not depend on $s$ and $\varphi_s(s,\theta) \equiv 0$. Thus, the first-order condition (3) collapses to:

$$k_M(\theta) = \frac{\theta}{c}. \tag{6}$$

Therefore, we have established:

**Proposition 1** *When the profitability parameter $\theta$ of the firm's investment is common knowledge and when the firm's investment is measured imprecisely, the equilibrium price in the capital market, $v(\frac{\theta}{c},\theta)$, is independent of the imprecise accounting measurement and the firm's equilibrium investment is $k_M(\theta) = \frac{\theta}{c}$.*

The intuitive reasoning underlying Proposition 1 is as follows: The equilibrium price in the capital market is based on an *anticipated* level of investment rather than the firm's actual investment. When the market observes a signal realization different from the anticipated investment, the market attributes the difference entirely to measurement noise, and therefore has no reason to revise its beliefs. Therefore, if the firm does depart from the market's anticipation of its investment, there is no change in the equilibrium market price, even though the distribution of the accounting signal does change. The firm responds to this situation by choosing its investment to maximize only its short-term return. Since such myopic investment is optimal for the firm regardless of the anticipated investment that is incorporated in the equilibrium market price, the *only* rational (sustainable) anticipation by the market is that the firm will indeed invest myopically. The equilibrium market price reflects this rational anticipation.

Proposition 1 is a very stark result. The accounting signal is completely ignored by the capital market even though there is a well defined statistical relationship between the signal and the firm's investment. The firm's behavior is myopic; it maximizes only its short-term return of $\theta k - \frac{ck^2}{2}$. It follows that if the marginal, long-term return to investment is high, the magnitude of underinvestment is very large. This (extremely bad) equilibrium is inescapable whenever the market believes it can perfectly foresee the firm's investment.

In the real world, imprecision in accounting measurements of investment are pervasive. We find it difficult to believe that such measurements have no information content and are ignored by the capital market. In addition, it seems unlikely that real investment in the economy exhibits the extreme myopia characterized in the above equilibrium. A possible explanation is that real world firm's do not seek to maximize their current market value, but we think such a behavioral hypothesis is implausible. Since there is no myopia in the full information setting, the problem here is not with value maximization *per se*, or with an over-emphasis on short-term vs. long-term value, but rather, the problem lies in the assumed information structure in the capital market[2]. This may suggest that the market has been assumed to know too little. However, we think the most plausible explanation is that the market has been assumed to know too much! The myopia here is due to the assumption that investors in the capital market can step into the manager's shoes and solve the manager's investment problem. It is this perfect anticipation of the firm's investment that is unrealistic and indefensible.

It seems realistic that corporate managers would have superior information about the profitability of new investment projects – at least at the time they are initiated. Screening alternative projects, assessing the future demand for new products, making cost and revenue projections, anticipating the retaliatory moves of competitors, and making judgments about future technological innovations are all tasks that have deliberately been delegated by shareholders to corporate managers presumably for informational advantages. If managers do possess such firm specific information that is not directly available to the capital market, then perfect anticipation of the firm's investment is no longer possible. We will show that in such asymmetric information environments noisy measurements of the firm's investment do have information content and do affect equilibrium capital market prices. In fact, given that lack of information asymmetry results in unrealistic myopia, it is difficult to justify the study of accounting imprecision in settings without asymmetric information.

[2]The claim that it is the assumed information structure that is driving myopic investment, rather than the assumed objective function, is reinforced by our results in Section 4. We show in Section 4 that a simple change in information structure leads to significant overinvestment rather than the underinvestment characterized here, even though the firm's objective function remains the same.

We model the manager's informational advantage by henceforth assuming the manager knows the parameter $\theta$ before choosing the firm's investment but, *a priori*, the market knows only the distribution, $h(\theta)$. Now, even though the market can anticipate the equilibrium investment schedule $k(\theta)$, the market cannot calculate the firm's actual investment because $\theta$ is unknown. Given the market's knowledge of the schedule $k(\theta)$ and the prior distribution $h(\theta)$, there is a well-defined prior distribution on $k$. Accounting measurements of investment will be used to update this prior distribution, thereby making the equilibrium market price a nontrivial function of the accounting measure. Thus, even though accounting is imprecise, the accounting signal would have information content. This suggests that some ignorance in the capital market about project profitability could actually be beneficial.

## 4 Ignorance Without Imprecision: A Signaling Equilibrium

Before we study the role of imprecise accounting measurements in asymmetric information settings, it is insightful to study how perfect measurements of investment will affect the equilibrium when the firm's manager privately knows the value of $\theta$. This corresponds to the case where there is ignorance in the market, but no imprecision in accounting measurements. We will establish that perfection in accounting measurements are extremely undesirable in such settings.

Suppose, now, that the manager privately observes $\theta$ before choosing the firm's investment and that the accounting system perfectly measures and reports that chosen investment. In this case, the equilibrium price in the capital market is a function only of the firm's investment, say $\varphi(k)$. Because the manager chooses investment in the light of his private information, the market would seek to make inferences about project profitability from the perfectly measured investment. These inferences are embedded in the equilibrium pricing schedule $\varphi(k)$. Because measurement is perfect, there is the possibility that the market's inference of $\theta$ is also perfect, resulting in a classic signaling equilibrium similar to Spence (1974) and others. We show, below, that such a fully revealing signaling equilibrium is indeed sustainable and we analyze its properties.

**Definition 2** *A fully revealing signaling equilibrium consists of three schedules:*

$k(\theta)$ = *the firm's investment schedule,*

$\varphi(k)$ = *the capital market's pricing rule, and*

$I(k)$ = *an inference schedule,*

*that satisfy*

*(i)* $k(\theta) = \arg\max_k \ \theta k - \frac{ck^2}{2} + \varphi(k)$,

*(ii)* $\varphi(k) = v(k, I(k))$, *and*

*(iii)* $I(k(\theta)) = \theta$, $\forall\theta$.

Condition (i) requires that the equilibrium investment schedule maximizes the firm's payoff, given the pricing rule in the capital market. Condition (ii) requires that each possible investment that could be chosen by the firm is priced consistent with the market's point inference of project profitability. Thus, when the accounting system reports an investment of $k$ and the market infers that the value of $\theta$ must be $I(k)$, the equilibrium price that must prevail in the market is $v(k, I(k))$. Condition (iii) requires that, in equilibrium, the market's inference of $\theta$ from each observed investment coincides with the value of $\theta$ that gave rise to that investment. This rational expectations condition requires invertibility of the equilibrium investment schedule.

We use the mechanism-design methodology to characterize investment schedules that are consistent with signaling equilibria.[3] If $k(\theta)$ is an equilibrium investment schedule, it must be the case that for any two types $\theta$, and $\hat{\theta}$ type $\theta$ prefers $k(\theta)$ to $k(\hat{\theta})$ and type $\hat{\theta}$ prefers $k(\hat{\theta})$ to $k(\theta)$. If additionally, $k(.)$ is a *fully revealing* equilibrium investment schedule, it must satisfy the following incentive compatibility conditions:

$$\theta k(\theta) - \frac{ck^2(\theta)}{2} + v(k(\theta), \theta) \geq \ \theta k(\hat{\theta}) - \frac{ck^2(\hat{\theta})}{2} + v(k(\hat{\theta}), \hat{\theta}), \ \forall \theta, \hat{\theta} \tag{7}$$

Conditions (ii) and (iii) of equilibrium are embedded in (7). Denote the left hand side of (7) by $\Omega(\theta)$, so that the incentive compatibility conditions can be expressed as,

$$\Omega(\theta) \geq \Omega(\hat{\theta}) - k(\hat{\theta})[\hat{\theta} - \theta], \ \forall \theta, \hat{\theta} \tag{8}$$

[3]The link between the Spence/Riley methodology of constructing signaling equilibria and the mechanism design approach used here is formalized explicitly in Kanodia and Lee (1998).

Analysis of (8) yields the following necessary and sufficient conditions for incentive compatibility.

**Lemma 1** *An investment schedule $k(\theta)$ satisfies (7) if and only if:*

*(i)* $\Omega'(\theta) = k(\theta), \forall\ \theta$, *and*

*(ii)* $k(\theta)$ *is increasing.*

**Proof.** See the Appendix. ∎

Lemma 1 can be used to characterize the equilibrium investment schedule in the form of a differential equation. Let the interval $\Theta \equiv [\underline{\theta},\ \overline{\theta}]$ be the support of the distribution of $\theta$. Then from (i) of Lemma 1 it follows that,

$$\int_{\underline{\theta}}^{\theta} \Omega'(t)dt = \int_{\underline{\theta}}^{\theta} k(t)dt$$

which implies that,

$$\Omega(\theta) = \int_{\underline{\theta}}^{\theta} k(t)dt + \Omega(\underline{\theta})$$

Using this with the definition of $\Omega(.)$ implies that an equilibrium investment schedule must satisfy,

$$\frac{ck^2(\theta)}{2} - \theta k(\theta) + \int_{\underline{\theta}}^{\theta} k(t)dt + \Omega(\underline{\theta}) = v(k(\theta), \theta) \tag{9}$$

Equation (9) should not be interpreted as a constraint on the market's pricing rule $v(.)$, which must be sequentially rational and market clearing, but rather as a condition on the equilibrium investment schedule. Differentiating (9)) with respect to $\theta$, yields:

**Proposition 2** *In a setting where the firm's manager privately observes $\theta$ before choosing investment, and investment is perfectly measured and reported by the accounting system, any equilibrium investment schedule must satisfy the monotonicity condition $k'(\theta) > 0$, and the first order differential equation,*

$$k'(\theta)[ck(\theta) - \theta - v_k] = v_\theta \tag{10}$$

*The firm overinvests at each $\theta > \underline{\theta}$.*

Riley (1979) shows that differential equations of this nature have a one parameter family of solutions, and that the exogenous parameter can be chosen so that the "worst" type invests the first-best quantity, in which case $k'(\theta) > 0$. Given $k'(\theta) > 0$ and $v_\theta > 0$, (10) can only be satisfied if $\theta - ck(\theta) + v_k < 0$, which implies that the firm overinvests, since first-best investment satisfies $\theta - ck(\theta) + v_k = 0$.

In a later section, we parametrically examine the magnitude of the overinvestment described in Proposition 2 and find that it can be very substantial. The extent of overinvestment at any value of $\theta$ does not depend on the prior distribution of $\theta$, so that even a very small probability mass on low values of $\theta$ shifts the entire investment schedule upwards. Overinvestment arises because to make inferences about the profitability parameter $\theta$ from the firm's observed investment, market participants must form beliefs about the firm's investment policy. How each observed investment is priced in the capital market depends strongly on these beliefs and inferences. Inferences based on the first-best investment schedule cannot be sustained because they lead to market prices which increase "too rapidly" in observed investment. Given such pricing, high levels of investment become so much more attractive, relative to low levels of investment, that low $\theta$ types choose investment levels that the market believes only high types would choose. Thus market participants would be systematically deceived and lose money, thereby inducing a revision in their beliefs. In equilibrium, beliefs shift in such a way that the market is no longer deceived, and equilibrium market prices are consistent with both the observed investment and its underlying profitability. However, the shift in beliefs that occurs due to the *possibility* of deception induces firms to overinvest and the cost of this overinvestment is born entirely by the firm's current shareholders.

Once again, the economy is trapped in a bad equilibrium. Now the firm is induced to overinvest, whereas previously it was optimal to underinvest. We now investigate the more realistic setting where there is both ignorance and imprecision, i.e., the manager is better informed than the capital market and the accounting measurement is imprecise. We show that both ignorance and imprecision are desirable, in the sense that they sustain more efficient equilibria.

## 5 Imprecision and Ignorance: A Noisy Signaling Equilibrium

Assume that the manager privately observes the profitability parameter $\theta$ before choosing the firm's investment and that investment is measured imprecisely by the accounting system. As before, the accountant's imprecise measurement system is represented by a probability density function, $f(s|k)$, where $s$ is the accounting measure and $k$ is the firm's true investment. Now, the price in the capital market can be a function only of $s$, say $\varphi(s)$. Embedded in this pricing rule are the market's inferences about the firm's investment and its profitability from observation of the accounting measure.

We have shown that when the market perfectly observes the firm's investment it can make a perfect inference of profitability, and when the market directly observes profitability it can make a perfect inference of the firm's investment. However, when both $\theta$ and $k$ are unobservable, the market's inference can no longer be perfect. Market inferences must take the form of a Bayesian posterior distribution on feasible values of $(k, \theta)$ conditional on $s$. Because, in equilibrium, the market can calculate the firm's investment policy, this posterior distribution reduces to a distribution on $\Theta$ conditional on $s$. If the market believes that the firm's investment schedule is $\hat{k}(\theta)$ then the assessed posterior distribution on $\Theta$, conditional on $s$ must satisfy:

$$g(\theta|s) = \frac{f\left(s|\hat{k}(\theta)\right) h(\theta)}{\int_{\Theta} f\left(s|\hat{k}(t)\right) h(t)\, dt}$$

In the above equation, $f\left(s|\hat{k}(\theta)\right)$ is the appropriate density at $s$ conditional on $\theta$ since the market believes that at $\theta$ the firm chooses investment of $\hat{k}(\theta)$.

**Definition 3** *An equilibrium is a triple $\langle k(\theta), g(\theta|s), \varphi(s)\rangle$ such that:*

*(i) Given $\varphi(s)$, $k(\theta)$ is optimal for the firm, i.e. $\forall\ \theta$, $k(\theta)$ solves*

$$\max\ \ \theta k - \frac{c}{2}k^2 + \int_s \varphi(s) f(s|k)\, ds,$$

*(ii) The market's beliefs are consistent with the equilibrium investment schedule of the*

*firm, i.e.,*

$$g(\theta|s) = \frac{f\left(s|k\left(\theta\right)\right)h\left(\theta\right)}{\int_{\Theta} f\left(s|k\left(t\right)\right)h\left(t\right)dt} \tag{11}$$

*(iii)* $\varphi\left(s\right)$ *is sequentially rational and market clearing, i.e.*

$$\varphi\left(s\right) = \int_{\Theta} v\left(k(\theta), \theta\right) g\left(\theta|s\right) d\theta, \tag{12}$$

The above definition describes a 'noisy' signaling equilibrium in the sense of Matthews and Mirman (1983). The firm's investment affects the *distribution* of a signal which is then priced in the market in accordance with the rational, but noisy, inferences made by the market. Unlike the perfect measurement case, (12) indicates that the equilibrium price in the market incorporates a pooling of types. However, unlike traditional notions of pooling where the weight on each type is defined by the prior distribution $h(\theta)$, here the weights are *equilibrium* weights which depend upon (i) the equilibrium investment schedule, (ii) the accounting measurement system, and (iii) the prior distribution of types. In a fully revealing signaling equilibrium, the prior distribution on types is immaterial. Here, the prior distribution affects the firm's investment through its effect on equilibrium capital market prices.

As in the perfect measurement case, the equilibrium investment schedule is characterized by the mechanism-design approach. Given a pricing rule, $\varphi(s)$, if $k(\theta)$ is an optimal investment schedule it must satisfy the incentive-compatibility conditions:

$$\begin{aligned} &\theta k\left(\theta\right) - \frac{c}{2}k^2\left(\theta\right) + \int_s \varphi\left(s\right) f\left(s|k\left(\theta\right)\right) ds \\ &\geq \theta k\left(\hat{\theta}\right) - \frac{c}{2}k^2\left(\hat{\theta}\right) + \int_s \varphi\left(s\right) f\left(s|k\left(\hat{\theta}\right)\right) ds \qquad \forall \theta, \hat{\theta}. \end{aligned} \tag{13}$$

Denoting the left hand side of (13) by $\Lambda(\theta)$, the above inequalities are equivalent to:

$$\Lambda\left(\theta\right) \geq \Lambda\left(\hat{\theta}\right) - k\left(\hat{\theta}\right)\left(\hat{\theta} - \theta\right) \tag{14}$$

Inequalities (13)) and (14) are identical to (7) and (8) except that the pricing rule $v(k(\theta), \theta)$ is replaced by $\int_s \varphi\left(s\right) f\left(s|k\left(\theta\right)\right) ds$. Hence, a result similar to Lemma 1 holds:

**Lemma 2** *An investment schedule* $k(\theta)$ *satisfies (13) if and only if:*

*(i)* $\Lambda'(\theta) = k(\theta), \ \forall \ \theta$ *and*

*(ii)* $k(\theta)$ *is increasing.*

**Proof.** Similar to the proof for Lemma 1 and, hence, is omitted. ∎

Using Lemma 2 in exactly the same way that Lemma 1 was used, we find that the investment schedule must satisfy:

$$\int_S \varphi(s) f(s|k(\theta)) ds = \frac{ck^2(\theta)}{2} - \theta k(\theta) + \int_{\underline{\theta}}^{\theta} k(t)dt + \Lambda(\underline{\theta})$$

Differentiating with respect to $\theta$ and canceling common terms yields the equivalent of the first-order condition to the firm's optimization program:

$$\int_S \varphi(s) f_k(s|k(\theta)) ds = ck(\theta) - \theta \tag{15}$$

In the above derivations, $\varphi(s)$ was an exogenous pricing rule that the firm takes as given. Therefore incentive compatibility was defined relative to $\varphi(s)$. In equilibrium, the pricing rule in the capital market must be consistent with the investment schedule that is incentive compatible relative to that pricing rule. This is the requirement that is captured in the equilibrium conditions (11) and (12). Inserting (11) and (12) into (15 ) provides the following characterization:

**Proposition 3** *In a setting where the firm's manager privately observes $\theta$ before choosing investment, and the investment is measured imprecisely (in accordance with the probability density function $f(s|k)$), any equilibrium investment schedule $k(\theta)$ must satisfy,*

$$\int_S \left\{ \int_\Theta v(k(t), t) \frac{f(s|k(t)) h(t)}{\int_\Theta f(s|k(\tau)) h(\tau) d\tau} dt \right\} f_k(s|k(\theta)) ds = ck(\theta) - \theta \tag{16}$$

The equilibrium that gives rise to myopic investment that was studied in Section 3, and the fully revealing equilibrium that gives rise to overinvestment that was studied in Section 4, are special cases of the more general equilibrium described in (16). Myopia occurs when the value of $\theta$ is publicly observed. Let $\theta^0$ be the observed value of $\theta$. Then, $\forall s$, $g(\theta|s) = 1$ if $\theta = \theta^0$, and $g(\theta|s) = 0$ if $\theta \neq \theta^0$. Given that all of the probability mass is on $\theta^0$, (12) implies that $\varphi(s) = v(k(\theta^0), \theta^0)$, $\forall s$. Thus (16) becomes,

$$\int_S v(k(\theta^0), \theta^0) f_k(s|k(\theta^0)) ds = ck(\theta^0) - \theta^0$$

Because $\int_S f_k(s|k)ds = 0, \forall k$, the above equation collapses to $ck(\theta^0) - \theta^0 = 0$ or myopic investment. Perfect measurement arises when $s \equiv k$. Let $K(\theta)$ be the equilibrium perfect-measurement investment schedule, and let $k$ denote an observed level of investment. Then the posterior density $g(\theta|k)$ is described by $g(\theta|k) = 1$ if $\theta = K^{-1}(k)$ and $g(\theta|k) = 0$ for all other values of $\theta$. Then, (12) becomes,

$$\varphi(k) = \int_{\Theta} v(K(\theta), \theta)\, g(\theta|k)\, d\theta = v(k, K^{-1}(k))$$

In this case (16) is equivalent to:

$$\frac{d}{dk}\left\{v(k, K^{-1}(k))\right\} = cK(\theta) - \theta,$$

which is equivalent to (10).

It is difficult to say anything about the equilibrium investment schedule for the general noisy signaling case, characterized in (16) without imposing some regularity conditions on the measurement rule $f(s|k)$. It seems natural to require that on average the accounting measure is higher when the firm's investment is higher, and that higher values of the accounting measure constitute good news, in the sense of Milgrom (1981). We model this by assuming that $f(s|k)$ has the monotone likelihood ratio property (MLRP), i.e., for any $k'' > k'$, $\frac{f(s|k'')}{f(s|k')}$ is strictly increasing in $s$. This assumption additionally implies that higher investment shifts the distribution of the accounting measure to the right in the sense of first-order stochastic dominance (FSD). Milgrom (1981) established that if $f(s|k)$ satisfies MLRP then the induced posterior distribution on investment conditional on the signal $s$ satisfies FSD for every prior distribution on investment. Combined with the equilibrium requirement that the investment schedule $k(\theta)$ is increasing, this implies that the posterior density $g(\theta|s)$ also satisfies FSD. Now, since $\varphi(s) = \int v(k(\theta), \theta)g(\theta|s)d\theta$ and $v$ is strictly increasing in $\theta$, $\varphi(s)$ is strictly increasing. In turn, this implies that $\int \varphi(s)\, f_k(s|k)ds = -\int \varphi'(s)F_k(s|k)ds > 0$. Using this fact together with the firm's first order condition for a maximum implies that any solution to the integral equation (16) must have the property that at each $\theta > 0$, the firm's equilibrium investment is greater than the myopic amount. This result corresponds to the obvious intuition that if noisy measurements of investment have any value at all, such measurements must induce the firm to investment beyond the myopic amount.

We do not prove existence of a solution to the general integral equation (16). Instead, we specialize the analysis to a parametric family of measurement rules and valuation rules for which we are able to obtain closed form solutions to (16). We then investigate how the equilibrium investment schedule and the equilibrium pricing rule in the market change as accounting measurement rules ($f(s|k)$), and prior beliefs ($h(\theta)$), become more or less precise.

## 6 Optimality of Imprecision and Ignorance

We make two assumptions to specialize the model. First, we assume that imprecise accounting measurement rules correspond to members of a parametric family of distributions. Second, we specialize the exogenous valuation rule $v(k,\theta)$ to a class for which linear investment schedules can be supported as equilibria. Specifically, we assume:

(A1) $\widetilde{s} = k + \widetilde{\epsilon}$, $\widetilde{\epsilon}$ is distributed normally with $E(\widetilde{\epsilon}) = 0,\ var(\widetilde{\epsilon}) = \sigma_\epsilon^2$

(A2) The prior distribution of $\widetilde{\theta}$ is normal with $E(\widetilde{\theta}) = \mu,\ var(\widetilde{\theta}) = \sigma_\theta^2$

(A3) $v(k,\theta) = \gamma\theta k + m\theta^2$, where $\gamma > 0$ and $m \geq 0$ are known constants.

Assumption (A1) requires accounting measurement rules to be unbiased and measurement errors to be normally distributed. Larger values of $\sigma_\epsilon^2$ correspond to less precise accounting measurement rules. Assumption (A2) has the unfortunate implication that optimal investments could become negative when the profitability parameter $\theta$ is sufficiently negative. We allow such negative investments to avoid truncating the distribution of $\theta$, though the interpretation of negative investment is problematic. Variations in the parameter $\sigma_\theta^2$ allows us to make the prior information about $\theta$ more or less precise, and increases in $\mu$ make prior beliefs more optimistic. Assumption (A3) says that in a complete information economy, where $\theta$ and $k$ are directly observed, the equilibrium valuation rule in the capital market has two components. The first component $\gamma\theta k$ represents the persistence in expected returns from the firm's current investment; where the parameter $\gamma$ could be interpreted as an earnings multiple or as the number of years of useful life of the project or as a present value factor. The second component $m\theta^2$, which does not depend on current investment, is intended to capture the effect of current profitability on the expected returns from anticipated *future* investment[4].

## 6.1 Characterizing Noisy Signaling Equilibria

Given (A3), the first-best investment schedule is:

$$k_{FB}(\theta) = \left(\frac{1+\gamma}{c}\right)\theta \tag{17}$$

The myopic investment schedule remains $k_M(\theta) = \frac{\theta}{c}$, as derived in (6). The fully revealing investment schedule, which would obtain when $\sigma_\epsilon^2 = 0$, can be derived from (10):

$$k'(\theta)[ck(\theta) - (1+\gamma)\theta] = \gamma k(\theta) + 2m\theta, \tag{18}$$

which admits the linear solution:

$$k_{PM}(\theta) = \left[\frac{1+2\gamma+\sqrt{(1+2\gamma)^2+8mc}}{2c}\right]\theta \tag{19}$$

Because all three of the above investment schedules are linear in $\theta$, we investigate the family of linear investment schedules as candidates for noisy signaling equilibria. Consider investment schedules of the form $k(\theta) = a + b\theta$, where $a$ and $b$ are endogenously determined constants. Given this linear investment schedule, the accounting measure is equivalent to:

$$\widetilde{s} = a + b\widetilde{\theta} + \widetilde{\epsilon}$$

so that the joint distribution of $(\widetilde{s}, \widetilde{\theta})$ is normal, and the conditional density $g(\theta|s)$ is also normal, with parameters:

$$\begin{aligned} E(\widetilde{\theta}|s) &= \mu + \frac{cov(\widetilde{\theta}, \widetilde{s})}{var(\widetilde{s})}\,[s - E(s)] \\ &= \mu + \frac{b\sigma_\theta^2}{b^2\sigma_\theta^2 + \sigma_\epsilon^2}\,[s - a - b\mu] \end{aligned}$$

Let,

$$\beta \equiv \frac{b^2\sigma_\theta^2}{b^2\sigma_\theta^2 + \sigma_\epsilon^2}$$

Then, if $b \neq 0$, the posterior mean can be expressed as:

$$E(\widetilde{\theta}|s) = (1-\beta)\mu + \beta\left(\frac{s-a}{b}\right) \tag{20}$$

[4] We use the square of $\theta$ to reflect the assumption that negative investment is feasible thus making negative values of $\theta$ similar to positive values of $\theta$.

and,

$$var(\widetilde{\theta}|s) = (1-\beta)\sigma_\theta^2 \tag{21}$$

Now, the left-hand side of (16) can be characterized in closed form. Inserting $k(\theta) = a + b\theta$, and $v(k,\theta) = \gamma\theta k + m\theta^2$ into the expression $\varphi(s) = E[(v(k(\theta),\theta)|s]$ gives,

$$\varphi(s) = [a\gamma]E(\theta|s) + [b\gamma + m]E(\theta^2|s) \tag{22}$$

Replacing $E(\theta|s)$ by (20) and using $E(\theta^2|s) = var(\theta|s) + [E(\theta|s)]^2$, where $var(\theta|s)$ is given by (21), yields the following quadratic expression for $\varphi(s)$:

$$\varphi(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2, \tag{23}$$

where,

$$\alpha_0 = a\gamma[(1-\beta)\mu - \frac{\beta a}{b}] + (b\gamma + m)[(1-\beta)\sigma_\theta^2 + \{(1-\beta)\mu - \frac{\beta a}{b}\}^2] \tag{24}$$

$$\alpha_1 = \frac{\beta}{b}[a\gamma + 2(b\gamma + m)\{(1-\beta)\mu - \frac{\beta a}{b}\}] \tag{25}$$

$$\alpha_2 = \frac{\beta^2(b\gamma + m)}{b^2} \tag{26}$$

Using (23) the left-hand side of (16) becomes:

$$\int_S \varphi(s)\, f_k(s|k(\theta))ds = \alpha_1 \int_S s f_k(s|k(\theta))ds + \alpha_2 \int_S s^2 f_k(s|k(\theta))ds$$

For the Normal density, we have

$$f_k(s|k) = f(s|k)\left[\frac{s-k}{\sigma_\epsilon^2}\right]$$

Therefore, for any $k$,

$$\begin{aligned}\int_S \varphi(s)\, f_k(s|k)ds &= \frac{\alpha_1}{\sigma_\epsilon^2}\left[E(s^2|k) - kE(s|k)\right] + \frac{\alpha_2}{\sigma_\epsilon^2}\left[E(s^3|k) - kE(s^2|k)\right] \\ &= \frac{\alpha_1}{\sigma_\epsilon^2}[\sigma_\epsilon^2 + k^2 - k^2] + \frac{\alpha_2}{\sigma_\epsilon^2}[k^3 + 3k\sigma_\epsilon^2 - k^3 - k\sigma_\epsilon^2] \\ &= \alpha_1 + 2\alpha_2 k\end{aligned}$$

The equilibrium condition, described by (16), becomes:

$$\alpha_1 + 2\alpha_2 k = ck - \theta.$$

Thus, the solution for $k$ has a linear form,

$$k(\theta) = \frac{\alpha_1}{c - 2\alpha_2} + \frac{1}{c - 2\alpha_2}\theta. \tag{27}$$

The second order condition for a maximum is satisfied if $c - 2\alpha_2 > 0$.

We began the analysis by *assuming* linear investment schedule of the form $k(\theta) = a + b\theta$. The coefficients, $\alpha_0$, $\alpha_1$ and $\alpha_2$ in the quadratic pricing rule derived earlier, depend on the values of the parameters $a$ and $b$. Therefore, sustainable linear investment schedules must satisfy,

$$b = \frac{1}{c - 2\alpha_2} \tag{28}$$

$$\begin{aligned} a &= \frac{\alpha_1}{c - 2\alpha_2} \\ &= b\alpha_1. \end{aligned} \tag{29}$$

Equation (28) indicates that every sustainable value of $b$ that satisfies $b > 0$ must be such that $c - 2\alpha_2 > 0$ ensuring that the second order condition for a maximum is satisfied. In Lemma 3, below, we show that all sustainable linear investment schedules have $b > 0$. For every sustainable value of $b$, the values of $\beta$, $a$ and $\sigma_\varepsilon^2$, expressed as functions of $b$, are as follows. Inserting the value of $\alpha_2$,characterized in (26) into (28) and solving for $\beta$ yields,

$$\beta^2 = \frac{(bc-1)b}{2(b\gamma + m)}. \tag{30}$$

Inserting the value of $\alpha_1$, characterized in (25) into equation (29) yields,

$$a = \frac{2(b\gamma + m)\beta(1-\beta)\mu}{1 - \beta[\gamma - 2(b\gamma + m)\frac{\beta}{b}]}. \tag{31}$$

Now substituting $\beta \equiv \frac{b^2\sigma_\theta^2}{b^2\sigma_\theta^2 + \sigma_\epsilon^2}$ into equation (30) and solving for $\sigma_\varepsilon^2$ yields,

$$\sigma_\epsilon^2 = b^2\sigma_\theta^2\left[\sqrt{\frac{2(b\gamma + m)}{(bc-1)b}} - 1\right]. \tag{32}$$

To construct a sustainable linear investment schedule, first choose a sustainable value of $b$ (as specified in Lemma 3 below), then solve for $\beta$ from equation (30), then solve (31) for the value of $a$ that is implied by these values of $b$ and $\beta$, then determine $\sigma_\epsilon^2$ from (32).

The sustainable values of $b$ are determined from the requirement that $\sigma_\epsilon^2 \geq 0$. As $b \to \frac{1}{c}$, $\beta \to 0$, $\sigma_\epsilon^2 \to \infty$, and $a \to 0$. This corresponds to the myopic investment schedule.

As $b \to b_{PM} = \frac{1+2\gamma+\sqrt{(1+2\gamma)^2+8mc}}{2c}$, it can be verified that $2(b\gamma + m) \to (bc-1)b$, implying that $\sigma_\varepsilon^2$ as characterized in (32) converges to zero. This corresponds to perfect measurement, yielding the fully revealing signaling equilibrium characterized in (19).

**Lemma 3** *Linear investment schedules, $k(\theta) = a + b\theta$, that are sustainable as equilibria are those that satisfy:*

$$b \in \mathcal{B} \equiv \left\{ b \in R^+ \mid \frac{1}{c} \le b \le \frac{1 + 2\gamma + \sqrt{(1+2\gamma)^2 + 8mc}}{2c} \right\}, \tag{33}$$

*with $a$ defined by (30) and (31). The noise $\sigma_\epsilon^2$ that is needed to sustain any $b \in \mathcal{B}$ is characterized in (32).*

The procedure described above for construction of a sustainable investment schedule yields unique values for $a, \beta$,and $\sigma_\epsilon^2$ as functions of $b$. Unfortunately, the relationship between $b$ and $\sigma_\epsilon^2$ is not one-to-one. For those values of $\sigma_\epsilon^2$ that correspond to multiple values of $b$, there are multiple equilibria.

## 6.2 Complimentarity of Imprecision and Ignorance

We have characterized the linear investment schedules that can be supported as equilibria through the choice of imprecision in accounting measurements. Is there an *optimal* degree of imprecision? How much of an improvement do imprecise accounting measurements provide relative to the perfect measurement equilibrium? How does the optimal degree of imprecision in accounting measurements depend on the initial degree of information asymmetry between the firm's manager and the capital market? In this section we study these issues and provide some surprising answers (vis-a-vis conventional wisdom).

We study the properties of optimal accounting measurements from an *ex ante* perspective, i.e., we assume that accounting measurement rules are chosen before the manager has observed the firm's profitability $\theta$. An optimal measurement rule, $f^*(\cdot|\cdot)$, is one that maximizes the expected payoff of the firm's current shareholders, i.e.,

$$f^* \in \arg\max_{f(\cdot|\cdot)} E_\theta \left[ \theta k(\theta) - \frac{ck^2(\theta)}{2} + E_s(\varphi(s)|\theta) \right]$$

where, for each $f(\cdot|\cdot)$, $k(\theta)$ and $\varphi(s)$ are the equilibrium schedules implied by $f(\cdot|\cdot)$. The objective function can be simplified by using the law of iterative expectations[5] on the last term:

$$\begin{aligned} E_\theta\left[E_s(\varphi(s)|\theta)\right] &= E_s(\varphi(s)) \\ &= E_s[E_\theta\{v(k(\theta),\theta)|s\}] \\ &= E_\theta[v(k(\theta),\theta)] \end{aligned} \tag{34}$$

An accounting policy maker choosing among alternative measurement rules, ought to be concerned with how these measurement rules affect the *expected* price in the capital market, rather than the price response to specific realizations of $s$ and $\theta$. The derivation in (34) establishes that measurement rules affect the equilibrium expected price *only* through its effect on the equilibrium investment schedule of the firm. An investment policy that is optimal and sustainable under one measurement rule may not be sustainable under a different measurement rule. We think this is a general phenomenon, rather than an artifact of the particular model under study, and that a discussion of accounting policy without the explicit consideration of such *real* effects, is vacuous.

Because the first-best investment schedule maximizes $\theta k(\theta) - \frac{ck^2(\theta)}{2} + v(k(\theta),\theta)$ at each $\theta$, it follows from the above analysis that if there is an accounting measurement rule that sustains the first-best investment schedule in equilibrium, then that measurement rule is optimal. The next proposition characterizes such measurement rules given assumptions (A1) through (A3).

5

$$\begin{aligned} E_\theta\left[E_s(\varphi(s)|\theta)\right] &\equiv \int\left\{\int \varphi(s)\, f(s|k(\theta))\, ds\right\} h(\theta)\, d\theta \\ &= \int \varphi(s)\left\{\int f(s|k(\theta))\, h(\theta)\, d\theta\right\} ds \\ &= \int\left\{\int v(k(\theta),\theta)\, \frac{f(s|k(\theta))\, h(\theta)}{\int f(s|k(t))\, h(t)\, dt} d\theta\right\} \int f(s|k(\theta))\, h(\theta)\, d\theta ds \\ &= \int\int v(k(\theta),\theta)\, f(s|k(\theta))\, h(\theta)\, d\theta ds \\ &= \int v(k(\theta),\theta)\, h(\theta) \int f(s|k(\theta))\, ds d\theta \\ &= \int v(k(\theta),\theta)\, h(\theta)\, d\theta\,, \qquad \text{since} \int f(s|k)\, ds = 1,\ \forall k \end{aligned}$$

**Proposition 4** *The first-best investment schedule is sustainable if and only if $\mu \equiv E(\theta) = 0$. In this case, the optimal measurement rule is characterized by:*

$$\sigma_\epsilon^2 = \sigma_\theta^2 \left(\frac{1+\gamma}{c}\right)^2 \left[\sqrt{2\left(1+\frac{mc}{\gamma(1+\gamma)}\right)} - 1\right] \tag{35}$$

**Proof.** The first-best investment schedule is $k_{FB}(\theta) = a_{FB} + b_{FB}\theta$, where $a_{FB} = 0$, $b_{FB} = \left(\frac{1+\gamma}{c}\right)$. The linear investment schedules $k(\theta) = a + b\theta$ that can be sustained in equilibrium are characterized in Lemma 3. From (31) it is clear that $a = a_{FB} = 0$ if and only if $\mu = 0$ or $\beta = 0$ or $\beta = 1$. But as shown earlier, $\beta = 0$ results in myopic investment and $\beta = 1$ results in the fully revealing investment schedule. Therefore, $\mu = 0$ is necessary to sustain the first best investment schedule. With $\mu = 0$, $a = 0$ is the only self fulfilling conjecture by the market; regardless of the value of $\sigma_\epsilon^2$. Therefore $\sigma_\epsilon^2$ can be chosen solely to optimize the slope, $b$, of the investment schedule. Equation (32) characterizes the value of $\sigma_\epsilon^2$ that sustains feasible values of $b$. Inserting $b = b_{FB}$ in (32) and solving for $\sigma_\epsilon^2$ gives the desired result. ∎

The intuition underlying Proposition 4 is as follows. The firm's incentive to invest depends on the sensitivity of its capital market price, $\varphi(s)$, to the accounting measure. The more rapidly $\varphi(s)$ increases the more the firm would want to invest at any $\theta$. Given that the market price has the quadratic form $\varphi(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2$, the sensitivity of the price to the accounting measure is described by $\varphi'(s) = \alpha_1 + 2\alpha_2 s$. The component $\alpha_1$ that is independent of the accounting measurement provides a common incentive for investment, and therefore the value of $\alpha_1$ determines the firm's choice of $a$ in the linear investment schedule $k(\theta) = a + b\theta$. In turn, the sensitivity of the market price to the accounting measurement depends on the assessed posterior density $g(\theta|s)$. Holding the imprecision $\sigma_\epsilon^2$ of the measurement system fixed, this posterior density depends entirely on the market's conjecture, $\widehat{a}+\widehat{b}\theta$, of the firm's investment schedule. As shown in (20) higher values of $\widehat{a}$ shift the density to the left, decreasing $\varphi'(s)$ at every $s$, thereby decreasing the firm's incentive to invest. Now, consider the case $\mu > 0$. Suppose the market's conjecture has $\widehat{a} = 0$, and the price schedule $\varphi(s)$ is based on this conjecture. The value of the coefficient $\alpha_1$embedded in this price schedule is, $\alpha_1 = \frac{2(b\gamma+m)}{b}\beta(1-\beta)\mu > 0$, as calculated from (25 ). The firm would respond to such a pricing rule by choosing $a > 0$, as derived in (27), disconfirming the market's conjecture. This implies that the posterior distribution $g(\theta|s)$ that is assessed by the market lies to the right of the equilibrium distribution and the conjecture $\widehat{a} = 0$ cannot be sustained. This is why the first best investment investment schedule cannot be sustained when $\mu > 0$. Now consider the case where $\mu = 0$. The value of $\alpha_1$that is embedded in the market's pricing rule is $\alpha_1 = \widehat{a}\frac{\beta}{b}[\gamma - 2(b\gamma + m)\frac{\beta}{b}]$, where $\widehat{a}$ is the market's conjecture of the equilibrium $a$. Suppose $\beta$ is sufficiently big so that $\gamma - 2(b\gamma + m)\frac{\beta}{b} < 0$ (the intuition is easiest to see for such $\beta$). Now, if $\widehat{a} > 0$, $\alpha_1 < 0$, which decreases the firm's incentive to invest. The firm responds by choosing $a < 0$, disconfirming the market's conjecture. If $\widehat{a} < 0$, $\alpha_1 > 0$, increasing the firm's incentive to invest. The firm responds by choosing $a > 0$, once again disconfirming the market's conjecture. In fact, $\widehat{a} = 0$ is the only self fulfilling conjecture by the market when $\mu = 0$.

Since the firm's manager observes $\theta$ before choosing the firm's investment while the capital market does not, the parameter $\sigma_\theta^2$ can be interpreted as the degree of information asymmetry between the manager and the market (or the extent of the market's ignorance) regarding the profitability of the firm's investment. It might appear that the greater this information asymmetry the more precision one would like to build into the accounting measurement (if feasible). However (35) in Proposition 4 indicates the surprising result that the opposite is true. We formalize this result below.

**Corollary to Proposition 4** *When first-best investment is sustainable, the greater is the information asymmetry between the firm's manager and the capital market regarding the profitability of the firm's investment, the lower should be the precision with which the firm's investment is measured.*

The intuition for this result is as follows. When $\mu = 0$ (as required for sustainability of the first best investment schedule), $a = 0$, implying that $\alpha_0 = \alpha_1 = 0$, so the price in the capital market is $\varphi(s) = \alpha_2 s^2$. In this case, the sensitivity of the market price to the accounting signal is described by the coefficient $\alpha_2$, and (26) indicates that $\alpha_2$ is strictly increasing in $\beta$. Now when the market's prior information about project profitability is very precise, i.e. $\sigma_\theta^2$ is small, the market has little reason to revise its beliefs and therefore assigns a very low weight to the accounting signal. Thus, $\beta$ is low and the market price is relatively insensitive to the accounting signal. This induces the firm to invest myopically. In this case, it is desirable to increase the precision of the accounting measurement, thus increasing $\beta$

and moving the firm away from myopia. Conversely, when the market's prior information about project profitability is very imprecise, the weight assigned to the accounting signal is large making the market price very sensitive to the accounting signal which, in turn, induces the firm to overinvest. In this case it is desirable to decrease the precision of the accounting signal thus decreasing the value of $\beta$ and inducing the firm to reduce its investment. Attainment of the first-best investment schedule requires the weight $\beta$ to be just right, requiring an optimal ratio of $\sigma_\epsilon^2$ to $\sigma_\theta^2$ as indicated in (35).

The above results seem consistent with current accounting practice. A firm's financial statements, under GAAP, convey more precise information about the firm's investment in "property, plant and equipment" than its investment in R&D. Even though a firm's total expenditures on R&D are reported, no attempt is made to distinguish between productive R&D investment and worthless R&D expenses, resulting in high imprecision in individual investor attempts to disentangle them. At the same time, it is likely that there is more information asymmetry between the market and the firm's managers regarding the future returns, or market value of productive R&D, than of "property, plant and equipment."

The result in (35) also sheds light on a firm's disclosure policy regarding the profitability of its investment. Suppose the imprecision in reporting the firm's investment is beyond the manager's control, and the manager has the opportunity to *ex ante* commit to a reporting policy that credibly reveals information about the profitability parameter $\theta$. Our results indicate that the manager should not commit to fully reveal his information. If $\sigma_\theta^2$ is interpreted as the *posterior* variance of the market's assessed distribution of $\theta$ conditional on the information released by the manager, then (35) indicates that the more noise there is in measurement of investment, the less information the manager should reveal about the profitability of investment. For any amount of measurement imprecision, there is a unique, optimal level of ignorance about profitability.

It is difficult to analytically characterize the optimal imprecision in measurement rules when the first-best investment schedule cannot be sustained (i.e., when $\mu \neq 0$). However, we show that some degree of imprecision in accounting measurement is *always* desirable.

The problem of finding the optimal precision of accounting measurement is equivalent to searching over all sustainable (linear) investment schedules of the form $k(\theta) = a + b\theta$

to maximize the firm's expected payoff. Having found the optimal pair $\{a, b\}$ from the sustainable set, one can calculate the corresponding value of $\sigma_\epsilon^2$ from (32). In Lemma 3, we characterized the sustainable set of $\{a, b\}$ pairs as those that satisfy $b \in \mathcal{B}$ and $a = a(b)$, where $a(b)$ is defined by (30) and (31). Therefore, the optimal sustainable investment schedule, $a^* + b^*\theta$, is characterized by:

$$b^* \in \arg\max_{b \in \mathcal{B}} U(b) \equiv E_\theta \left[\theta(a(b) + b\theta) - \frac{c}{2}(a(b) + b\theta)^2 + \gamma\theta(a(b) + b\theta) + m\theta^2\right] \quad (36)$$

Evaluating the expectation with respect to $\theta$ and collecting terms, yields:

$$U(b) = [(1+\gamma)b - \frac{c}{2}b^2 + m](\sigma_\theta^2 + \mu^2) + (1 + \gamma - bc)a\mu - \frac{c}{2}a^2 \quad (37)$$

Differentiating with respect to $b$,

$$U'(b) = (1 + \gamma - bc)[\sigma_\theta^2 + \mu^2 + \mu\frac{\partial a}{\partial b}] - ca(\mu + \frac{\partial a}{\partial b}) \quad (38)$$

It is difficult to characterize the optimal value of $b$ because the function $a(b)$ is ill-behaved; there can be multiple local maxima. However, we show that $\lim U'(b)$ as $b \to b_{PM}$ is negative; thus, the firm's expected payoff is increasing away from $b_{PM}$. Because $b = b_{PM}$, is sustainable if and only if $\sigma_\epsilon^2 = 0$, every lower value of $b$ requires corresponding positive imprecision, i.e., $\sigma_\epsilon^2 > 0$. This establishes:

**Proposition 5** *If there exists information asymmetry between the firm's manager and the capital market regarding the profitability of investment, some degree of imprecision in accounting measurements of investment is always optimal.*

**Proof.** See the Appendix. ∎

While Proposition 5 establishes that *some* degree of imprecision is always desirable, it does not characterize the optimal level. To obtain insights into the relationship between the optimal level of imprecision and our exogenous variables we analyze the problem numerically. In addition, we illustrate the extent of improvement over perfect measurement. The results are surprising. We solve for the optimal level of imprecision for 14,320 combinations of the parameters, $\mu$, $\sigma_\theta^2$, $\gamma$, $m$ and $c$. On average perfect measurement attains 16% of the first best expected payoff, whereas the optimum choice of imprecision attains 96.8% of the first best expected payoff. We also find that the relationship between the optimal level of imprecision and the information asymmetry between the manager and the market, characterized in Proposition 4, continues to hold even when $\mu > 0$; the higher the level of information asymmetry, the higher is the optimal level of imprecision.

We investigated the following parameter values:

$$\mu \in \{2, 4, \ldots, 18, 20\}$$
$$\sigma_\theta = \left\{ \frac{1}{18}\mu, \frac{2}{18}\mu, \ldots, \frac{6}{18}\mu \right\}$$
$$\gamma \in \{2, 4, \ldots, 18, 20\}$$
$$m \in \{0, 2, \ldots, 10\}$$
$$c \in \{1, 2, \ldots, 5\}.$$

There are 15,000 combinations of parameters. For each combination we calculated the optimal value of $b$ and the corresponding value of $\sigma_\varepsilon^2$, using the Nelder-Mead type simplex search method to maximize the firm's expected payoff. Of these 15,000 the algorithm did not converge for 680 combinations and were consequently dropped. Given the optimal values of $b$ we calculated the following statistic:

$$T = \frac{U(b^*) - U(b_{PM})}{U_{FB} - U(b_{PM})},$$

where $U(b^*)$ is the firm's expected payoff (36) calculated under the optimal level of imprecision, $U(b_{PM})$ is the expected payoff under perfect measurement and $U_{FB}$ is the first best expected payoff. The statistic $T$ measures the fraction of the loss in expected payoff due to perfect measurement that is recovered by optimal imprecise measurement. The value of $T$ must lie between zero and one, where $T = 0$ indicates that perfect measurement is optimal and $T = 1$ indicates that $b^*$ implements the first best. In Proposition 4 we established that when $\mu = 0$, $T = 1$ for all $\sigma_\theta^2$, $\gamma$, $m$ and $c$. Figure 1 is a histogram of $T$. Notice that the minimum $T$ is 90.1% and the maximum $T$ is 99.9%. On average the optimal choice of imprecision recoups 96.2% of the efficiency loss associated with perfect measurement. Given the high minimum value of $T$ the standard deviation is only 2.7%. As can be seen from the histogram, the distribution is skewed to the right; the median value of $T$ is 97.04% and 20% of the cases provide improvements of greater than 98.86%.

Figure 2 illustrates the effect of an increase in the level of information asymmetry –

investor ignorance – on the optimal imprecision of the accounting measure. Consistent with Proposition 4, the optimal imprecision increases linearly in the level of information asymmetry. We expect this relationship will hold for other parameter values as well. Figure 2 also indicates a positive relationship between the optimal imprecision and the capitalization factor $\gamma$. This is consistent with the fact that the overinvestment induced under perfect measurement increases in $\gamma$, and thus the level of imprecision required to counteract the firm's incentive is higher.

## 7 Conclusion

There seems to be a belief among practitioners and financial researchers that accounting measurements should be made as precise as possible, and that any observed imprecision is to be tolerated as a necessary evil arising from limitations in measurement technology. Our results contradict this conventional wisdom. We have argued that, in market settings, a serious study of the economic consequences of imprecision can only be conducted in settings where (i) the firm's managers have information superior to the market regarding the environment in which managerial decisions are made, and (ii) the firm's managers are concerned with how their decisions are priced in the capital market. We find that in the presence of the information asymmetry, described in (i), very precise accounting measurements actually destroy value and adversely affect the payoffs to the firm's current shareholders. Imprecision in accounting measurements induce significantly better equilibria, and the optimal degree of imprecision is strictly increasing in the extent of information asymmetry between the manager and the capital market. Conversely, given imprecision in accounting measurement, it is highly desirable that mangers retain some information superiority over the capital market. In this sense, ignorance supports imprecision and imprecision supports ignorance. An appropriate mix of ignorance and imprecision produces consequences that are reasonably close to first best.

Our findings should be tempered by some of our assumptions that may not hold in real world settings. We have assumed that capital market participants have no opportunity to augment accounting information through private information search. If such search opportunities exist but are unequal across individuals, then imprecision in accounting measurements would allow the ‘privileged few’ to gain an informational advantage over the average investor. This may be socially undesirable. We have additionally assumed that the support of the accounting signal is independent of the true investment of the firm. Perhaps, real world measurements exhibit moving support. In fact, some would argue that, given conservatism in measurement rules, accounting reports of investment reflect the lower bound of the firm’s true investment. We have not investigated such moving support cases because they give rise to difficult issues concerning off equilibrium beliefs. Finally, we have assumed aggregate risk neutrality for the capital market. If, instead, there is aggregate risk aversion in the capital market, imprecision in accounting reports would increase the risk premium in the equilibrium capital market price, decreasing the benefits to imprecision. Investigation of these issues would enrich the understanding of the costs and benefits of imprecision in accounting measurements.

## Appendix

**Proof of Lemma 1:** To show the necessity we can write (8) as,

$$\Omega(\theta) - \Omega(\hat{\theta}) \geq -k(\hat{\theta})[\hat{\theta} - \theta], \ \forall \theta, \hat{\theta}. \tag{39}$$

Similarly, from the reverse IC condition of type $\hat{\theta}$ we obtain,

$$\begin{aligned} &\Omega(\hat{\theta}) - \Omega(\theta) \geq -k(\theta)[\theta - \hat{\theta}], \ \forall \theta, \hat{\theta} \\ &\Leftrightarrow \Omega(\theta) - \Omega(\hat{\theta}) \leq k(\theta)[\theta - \hat{\theta}], \ \forall \theta, \hat{\theta}. \end{aligned} \tag{40}$$

Combining (39) and (40) and dividing by $\left(\theta - \hat{\theta}\right)$ we obtain,

$$k(\theta) \geq \frac{\Omega(\theta) - \Omega(\hat{\theta})}{\theta - \hat{\theta}} \geq k\left(\hat{\theta}\right), \ \forall \theta > \hat{\theta}.$$

$$k(\theta) \leq \frac{\Omega(\theta) - \Omega(\hat{\theta})}{\theta - \hat{\theta}} \leq k\left(\hat{\theta}\right), \ \forall \theta < \hat{\theta}.$$

From the above two expressions it is easy to see that $k(\theta)$ is increasing and, therefore, continuous almost everywhere. In the limit as $\hat{\theta} \to \theta$, we have,

$$\Omega'(\theta) = k(\theta), \ \forall \theta, \text{ a.e.}$$

To show sufficiency consider the following,

$$\begin{aligned} \Omega(\theta) - \Omega\left(\hat{\theta}\right) &= \int_{\hat{\theta}}^{\theta} \Omega'(t)\, dt, \ \forall \theta, \hat{\theta} \\ &= \int_{\hat{\theta}}^{\theta} k(t)\, dt, \ \forall \theta, \hat{\theta} \\ &\geq k\left(\hat{\theta}\right) \int_{\hat{\theta}}^{\theta} dt, \ \forall \theta > \hat{\theta}, \text{ since } k(\cdot) \text{ is increasing.} \\ &= -k\left(\hat{\theta}\right)\left[\hat{\theta} - \theta\right], \ \forall \theta > \hat{\theta}, \end{aligned}$$

which is the IC condition in (8). Similarly for $\theta < \hat{\theta}$, consider,

$$\begin{aligned} \Omega\left(\hat{\theta}\right) - \Omega(\theta) &= \int_{\theta}^{\hat{\theta}} k(t)\, dt, \ \forall \theta, \hat{\theta} \\ &\leq k\left(\hat{\theta}\right) \int_{\theta}^{\hat{\theta}} dt, \ \forall \theta < \hat{\theta} \\ &= k\left(\hat{\theta}\right)\left[\hat{\theta} - \theta\right], \ \forall \theta < \hat{\theta}. \end{aligned}$$

Multiplying by $-1$ we get,

$$\Omega(\theta) - \Omega\left(\hat{\theta}\right) \geq -k\left(\hat{\theta}\right)\left[\hat{\theta} - \theta\right], \ \forall \theta < \hat{\theta},$$

which again is the IC condition in (8). ∎

**Proof of Lemma 3:** Since incentive compatibility requires $k'(\theta) \geq 0$, sustainable values of $b$ must satisfy $b \geq 0$. Therefore, from *(32)* $bc - 1 \geq 0$, yielding the lower bound on $\mathcal{B}$. Now, since $\sigma_\varepsilon^2$ must be non-negative, (32) implies that sustainable values of $b$ are those that satisfy,

$$\sqrt{\frac{2(b\gamma + m)}{(bc-1)b}} - 1 \geq 0.$$

which is equivalent to,

$$N(b) \equiv b^2 c - (1 + 2\gamma)b - 2m \leq 0.$$

At $b = \frac{1}{c}$, $N(b) = -\frac{2(\gamma + mc)}{c} < 0$. Since $N'(b) = 2bc - (1 + 2\gamma)$, $N(b)$ is strictly decreasing at every $b < \frac{1+2\gamma}{2c}$ and strictly increasing at every $b > \frac{1+2\gamma}{2c}$. Therefore, $N(b) < 0$ over the interval $\left[\frac{1}{c}, \frac{1+2\gamma}{2c}\right]$. Now, $N(b_{PM}) = 0$ and $b_{PM} > \frac{1+2\gamma}{2c}$. Therefore $N(b) > 0$, $\forall\, b > b_{PM}$, implying that any $b > b_{PM}$ cannot be sustained by any feasible choice of $\sigma_\epsilon^2$. This completes the proof. ∎

**Proof of Proposition 5:** We earlier established that,

$$U'(b) = (1 + \gamma - bc)[\sigma_\theta^2 + \mu^2 + \mu\frac{\partial a}{\partial b}] - ca(\mu + \frac{\partial a}{\partial b}).$$

We wish to investigate $\lim U'(b)$ as $b \longrightarrow b_{PM}$. Now, as $b \longrightarrow b_{PM}$, $a \longrightarrow 0$ and $\beta \longrightarrow 1$. Therefore,

$$\lim\ U'(b) = (1 + \gamma - b_{PM}c)\left[\sigma_\theta^2 + \mu^2 + \mu\ \lim\{\frac{\partial a}{\partial b}\}\right] \tag{41}$$

To investigate $\frac{\partial a}{\partial b}$, from (31),

$$a = I(b, \beta(b)) \equiv \frac{2(m + \gamma b)\beta\mu(1-\beta)}{1 - \left(\gamma - 2(m + \gamma b)\frac{\beta}{b}\right)\beta},$$

where $\beta(b)$ is defined by,

$$\beta^2 = \frac{b^2 c - b}{2(b\gamma + m)}. \tag{42}$$

Then $\frac{\partial a}{\partial b} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial \beta}\frac{\partial \beta}{\partial b}$. Now, $\frac{\partial I}{\partial b} = 0$ when evaluated at $\beta = 1$, and

$$\frac{\partial I}{\partial \beta} = \frac{-2(m + \gamma b_{PM})\mu b_{PM}}{b_{PM}(1+\gamma) + 2m}.$$

Totally differentiating (42) yields,

$$2\beta \frac{\partial \beta}{\partial b} = \frac{2(m+\gamma b)(2bc-1) - 2\gamma(bc-1)b}{4(m+\gamma b)^2}.$$

Simplifying and evaluating at $\beta = 1, b = b_{PM}$ gives,

$$\frac{\partial \beta}{\partial b} = \frac{(2b_{PM}c - 1)m + b_{PM}^2 c\gamma}{4(m+\gamma b_{PM})^2}.$$

Therefore,

$$\lim \frac{\partial a}{\partial b} = \frac{-\mu b_{PM}}{b_{PM}(1+\gamma) + 2m} \frac{(2b_{PM}c - 1)m + b_{PM}^2 c\gamma}{2(m+\gamma b_{PM})}.$$

Inserting this expression in (41) gives,

$$\lim U'(b) = (1+\gamma - b_{PM}c)\left[\sigma_\theta^2 + \mu^2\{1 - \frac{b_{PM}}{b_{PM}(1+\gamma)+2m} \frac{(2b_{PM}c-1)m + b_{PM}^2 c\gamma}{2(m+\gamma b_{PM})}\}\right].$$

Since $1+\gamma - b_{PM}c < 0$, a sufficient condition for $\lim U'(b) < 0$ is:

$$2(b_{PM}(1+\gamma)+2m)(m+\gamma b_{PM}) - b_{PM}\left(2b_{PM}cm - m + b_{PM}^2 c\gamma\right) > 0.$$

Inserting $b_{PM} = \frac{1+2\gamma+\sqrt{(1+2\gamma)^2+8mc}}{2c}$ the above inequality reduces (after considerable simplification) to,

$$\frac{1}{2c^2}\left(\sqrt{(1+4\gamma+4\gamma^2+8cm)}\left(\gamma+2\gamma^2+cm\right) + \gamma + 4\gamma^2 + 4\gamma^3 + 6cm\gamma + cm\right) > 0,$$

which is obviously satisfied. This completes the proof. ∎

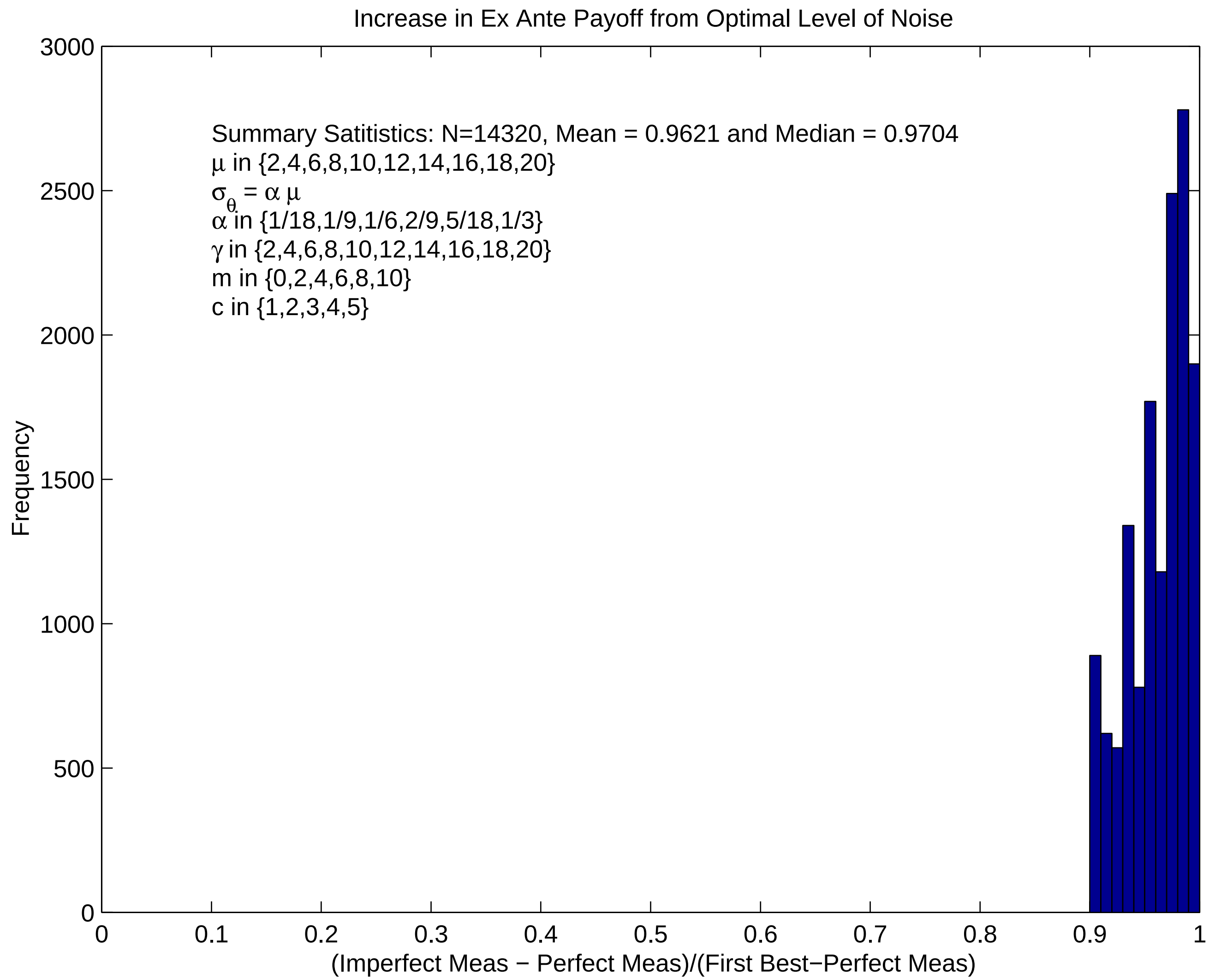


Figure 1: Improvement in Payoffs

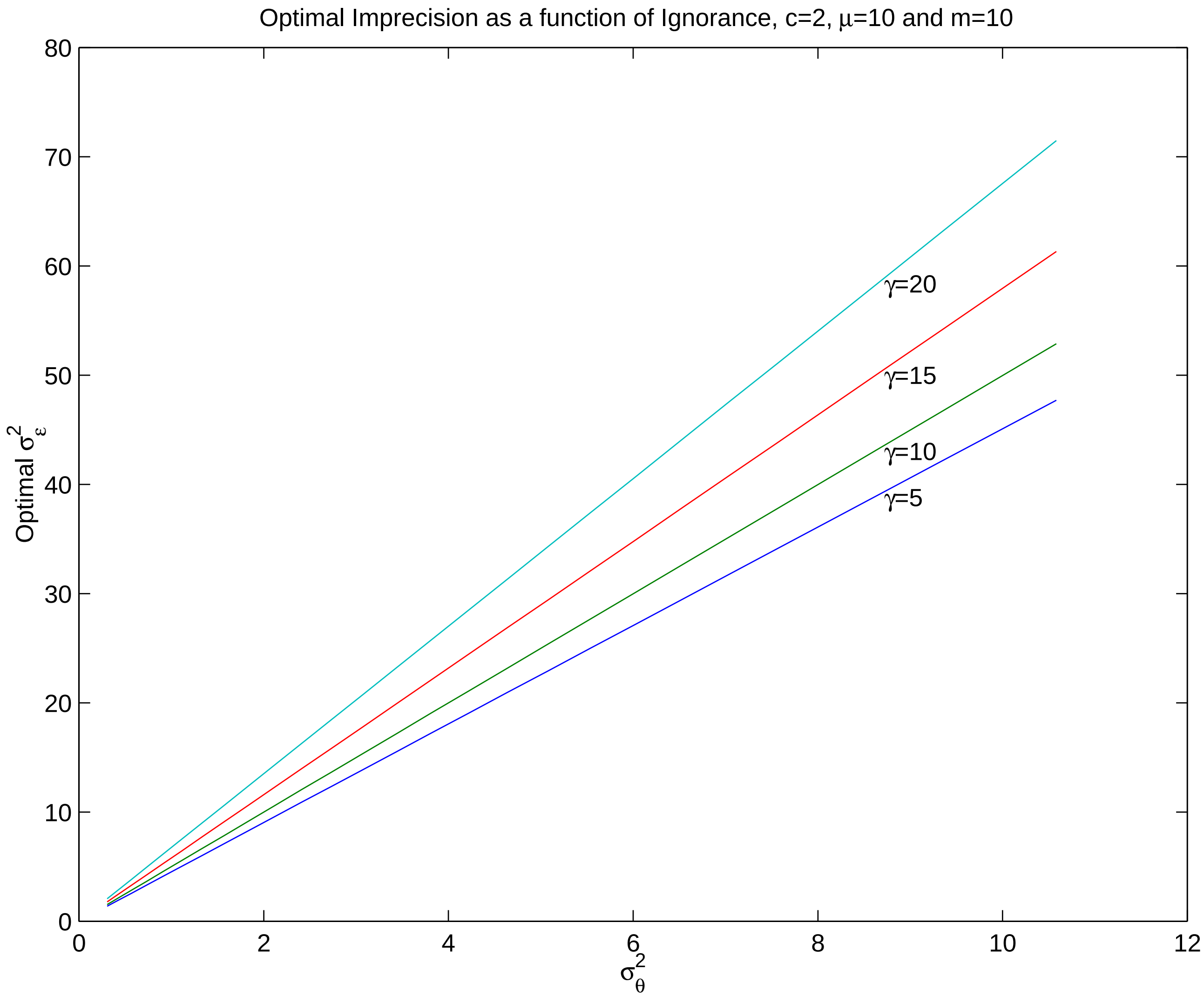


Figure 2: Optimal Imprecision of the Accounting Measure

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